Conference on Rings and Polynomials

Program

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Conference information

Date

July, 19 – 24, 2021

Venue

Graz University of Technology
Institute of Analysis and
Number Theory
Kopernikusgasse 24
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Austria

Scientific Comittee

Jean-Luc Chabert, Marco Fontana, Alfred Geroldinger, Sarah Glaz, Franz Halter-Koch, Irena Swanson

Local Organizers

Sophie Frisch, Amr Almaktry, Chimere Anabanti, Victor Fadinger, Sarah Nakato, Daniel Windisch

Bhargava's exponentials and Bhargava-Bernoulli numbers for primes

David Adam

Let E be a subset of $\mathbb{Z}$ and $(n!_E)_{n\in\mathbb{N}}$ be the sequence of Bhargava's factorials associated to E. Define the Bhargava's expoential of E by $\exp_{\mathcal{E}}(z) = \sum_{n\geq 0} \frac{z^n}{n!_E}$. Following Mingarelli, we will describe some irrationality and transcendence results about the values of $\exp_E$. We will introduce the analog of Bernoulli numbers for the Bhargava's exponential, and in the case of the set of prime numbers we will give some of their properties.

Maohi Nui University

An alternative perspective on divisible modules

Yusuf Alagöz

Some new studies in module theory have focused on to approach to the injectivity and flatness from the point of relative notions ([?, ?, ?]). In [?], Durg̃un is interested in the flat analog of these notions. Namely, given modules A_R and ${}_RB$, ${}_RB$ is called absolutely A_R-pure provided that for every extension ${}_RC$ of ${}_RB$, $A \otimes B \to A \otimes C$ is monomorphism. The class $\underline{\mathfrak{Fl}}^{-1}(A_R)$ $=\{{}_RB$: ${}_RB$ is absolutely A_R-pure$\}$ is called the absolutely pure domain of a module A_R (see [?]). As absolutely pure domains include all absolutely pure modules, the authors in [?] considered f-indigent modules as modules whose absolutely pure domain consists of entire class of absolutely pure modules.

If ${}_RB$ is divisible, then all short exact sequences starting with B is RD-pure, whence B is absolutey A-pure for every RD-flat module A_R. Thus for an RD-flat module A_R, the following relations are true: $\{$absolutely pure left modules$\}$ $\subseteq$ $\{$Divisible left modules$\}$ $\subseteq \underline{\mathfrak{Fl}}^{-1}(A) \subseteq$ R-mod. I will consider in this talk, the RD-flat modules whose absolutely pure domains contains only divisible modules, and we refered to these RD-flat modules as rd-indigent.

Various examples and properties of this concept are studied. In particular, for an arbitrary ring, the existence of the concept of rd-indigent modules is determined. It is also discussed the connections between rd-indigent and f-indigent modules. Then we give some characterizations of commutative semihereditary rings, Prüfer domains and von Neumann regular rings in terms of rd-indigent modules. In particuar, rd-indigent abelian groups are investigated. Some characterizations of the (commutative) rings whose non projective simple modules are rd-indigent are given.

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Polynomial functions of dual numbers of several variables

Amr Ali Abdulkader Al-Maktry

Let $k \in \mathbb{N} \setminus \{0\}$. For a commutative ring R, the ring of dual numbers of k variables over R is the quotient ring $R[x_1, \ldots, x_k]/I$, where I is the ideal generated by the set $\{x_i x_j : i, j = 1, \ldots, k\}$. This ring can be viewed as $R[\alpha_1, \ldots, \alpha_k]$ with $\alpha_i \alpha_j = 0$, where $\alpha_i = x_i + I$ for $i, j = 1, \ldots, k$. We investigate the polynomial functions of $R[\alpha_1, \ldots, \alpha_k]$ whenever R is a finite commutative ring. We derive counting formulas for the number of polynomial functions and polynomial permutations on $R[\alpha_1, \ldots, \alpha_k]$ depending on the order of the pointwise stabilizer of the subring of constants R in the group of polynomial permutations of $R[\alpha_1, \ldots, \alpha_k]$. Moreover, we prove that a function F on $R[\alpha_1, \ldots, \alpha_k]$ is a polynomial function if and only if a system of linear equations on R that depends on F has a solution.

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Pure descent for projectivity of modules

Gerhard Angermüller

In their seminal paper [1], Raynaud and Gruson asserted faithfully flat descent for projectivity . Shortly after, Gruson noted that the proof given there is not correct, but did not give any correction. In 2010 a proof was presented by Perry.
In the talk it is shown that results from [1], which are not affected by the above mentioned flaw, in fact prove faithfully flat descent. Further, a short (and easy to prove) complement even shows pure descent for projectivity.

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On n-pseudo-valuation domains

Ayman Badawi

Let R be an integral domain with quotient field K. A proper ideal I of R is an *n-powerful ideal* of R if whenever $x^n y^n \in I$ for $x, y \in K$, then $x^n \in R$ or $y^n \in R$; and I is an *n-powerful semiprimary ideal* of R if whenever $x^n y^n \in I$ for $x, y \in K$, then $x^n \in I$ or $y^n \in I$. If every prime ideal of R is an n-powerful semiprimary ideal of R, then R is an *n-pseudo-valuation domain* (*n-PVD*). In this talk, we study the above concepts and relate them to several generalizations of pseudo-valuation domains For more details, see [2].

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Complement-finite ideals

Nicholas Baeth

Let M be a (multiplicative) submonoid (with identity 1) of a commutative free monoid F such that $|F \setminus M| < \infty$ and $fm \in M$ for all $f \in F$ and all $m \in M \setminus \{1\}$. We call such a monoid a *complement-finite ideal of F*. Examples include $(S \setminus \{0\}) \cup \{1\}$, a complement-finite ideal of $\mathbb{N}$, when S is a numerical monoid and $\mathcal{F}(G)\mathcal{B}(G)^{\bullet}$, a complement-finite ideal of $\mathcal{F}(G)$, the monoid of formal sequences over an abelian group G which are not zero-sum free. We will consider various algebraic properties of these monoids; for example, they are never Krull except for in the trivial case, yet are always C-monoids. After classifying the irreducible elements of these atomic monoids we will give some preliminary results on their arithmetic; for example, every length set is an almost arithmetic progression with distance 1.

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Abundant Numbers and Semigroup Ideals in Factorial Monoids

Paul Baginski

A positive integer n is abundant if the sum of its divisors, $\sigma(n) = \sum_{d|n} d$, is greater than $2n$; n is perfect if $\sigma(n) = 2n$; and otherwise n is deficient. Both the set H of abundant numbers and the set H^* of non-deficient numbers are closed under multiplication, making them subsemigroups (in fact, semigroup ideals) of $(\mathbb{N}, \times)$. As a result, we can consider how elements of H^* (or H) factor into irreducible elements of H^* (resp. H), a concept related to Dickson's notion of primitive non-deficient numbers [1]. As it turns out, non-deficient numbers (or abundant numbers) do not factor uniquely into products of irreducible non-deficient numbers (resp. irreducible abundant numbers). We describe the factorization theory of these two semigroups, showing that they possess rather extreme factorization behavior. These two semigroups are special cases of semigroup ideals in factorial monoids. We shall describe the algebraic and arithmetic properties of such semigroup ideals. This will include a discussion of when such semigroup ideals are Krull monoids, C-monoids, or v-noetherian, as well as arithmetic conditions guaranteeing the extreme factorization behavior seen in abundant numbers.

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On the arithmetic of stable domains

Aqsa Bashir

A commutative ring R is stable if every non-zero ideal I of R is projective over its ring of endomorphisms $(I : I)$. Motivated by a paper of Bass in the 1960s, stable rings have received wide attention in the literature ever since then. It is well-known that every order in a quadratic number field is a stable domain. In this talk, we present the arithmetic of stable domains, with a focus on arithmetic properties of semigroups of ideals of stable orders in Dedekind domains. It is based on a joint work with Alfred Geroldinger and Andreas Reinhart.

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Nonnoetherian integral domains and the geometry of spacetime

Charlie Beil

I will present a geometric description of nonnoetherian subalgebras of affine coordinate rings, using a generalized notion of the height of an ideal. I will then show how this new geometry models a spacetime where time passes if and only if something changes, and describe certain quantum-like features that arise from such a 'nonnoetherian spacetime'. (No knowledge of physics will be assumed.)

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Specht's theorem, commutativity theorems, and decision procedures

Jason Bell

We consider the following question: Given a finite set of multivariate polynomial identities $P_1 = P_2 = \cdots = P_d = 0$, is it the case that a ring which satisfies these identities is necessarily commutative? As it turns out, one can use work related to Specht's theorem on affine representability to give a decision procedure which takes as input the set of identities and terminates after finitely many steps and gives an answer to this question. We then revisit old commutativity theorems of Jacobson and Herstein in light of this algorithm and obtain general results in this vein. This is joint work with Peter Danchev.

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Some remarks on polynomials with values which are powers of integers

Rachid BOUMAHDI

In 1913, Grosch proved that a polynomial $P(x) \in Z[x]$ whose integer values are squares is itself the square of another polynomial. In 1959, In turn, Shapiro proved that if P and Q are two polynomials with integer values of degrees p and q respectively, q divides p and $P(n) = Q(m)$ for an infinity blocks of integers of length $p/q + 2$, then $P(x) = Q(R(x))$ for some polynomial R.
In 2018, the author and J. Larone proved the following result: If $P(x)$ is a polynomial with integer coefficients of degree n and q a divisor of n.We suppose that there are M blocks of integers, with M sufficiently large, each containing $(n/q) + 2$ consecutive integers such that for all x in one of these blocks we have $P(x) = y^q$ for some y dependent on x. Then there exist a polynomial $R(x)$ with integer coefficients such that $P(x) = (R(x))^q$. In this talk we will discuss the previous result and some new questions concerning integer valued polynomials.
This will be a talk at the special session "Integer-valued polynomials".

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[2] Shapiro, H.S, The range of an integer-valued polynomial, Amer. Math. Monthly, 1957, 64

Functional Identities

Matej Brešar

The first, larger part of the talk will be a survey of the theory of functional identities and its applications [1]. The second part will present some connections between functional identities and the recently developed theory of zero product determined algebras [2].

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Operator-algebraic elements in prime rings

Jose Brox

Let A be a prime ring with extended centroid $\mathcal{C}$. Considering the Lie ring A^{-} with bracket product $[x, y] := xy - yx$ and $\mathrm{ad} : A \to A$ operator $\mathrm{ad}_x y := [x, y]$, an element $a \in A$ is called *ad-nilpotent* if there is $n \in \mathbb{N}$ such that $\mathrm{ad}_a^n = 0$ (as a map). By a theorem of Martindale and Miers, if the characteristic of A is good enough, an element is ad-nilpotent if and only if it is the sum of a nilpotent and a "central" element (element of the extended centroid), so it is algebraic with minimal polynomial $(X - \lambda)^m \in \mathcal{C}[X]$ for some $\lambda \in \mathcal{C}$ and $m \in \mathbb{N}$. More in general, for $a \in A$ let L_a, R_a denote the left and right multiplication operators respectively, and from any fixed polynomial in 2 variables $f \in \mathcal{C}[X, Y]$ define the operator $f_a := f(L_a, R_a)$ for each $a \in A$. We say that the element $a \in A$ is *f-algebraic* if $f_a = 0$ as a map. Martindale's lemma for prime rings shows that if f has no constant term then an f-algebraic element is algebraic, but its application is not effective, in that it does not explicitly provide its possible minimal polynomials. We determine, given f, the minimal polynomials that the f-algebraic elements can present. To do so we rewrite the problem as a question in ideals of polynomial rings in two variables, and then apply Gröbner bases at an elementary level, partial Hasse derivatives, and a multiset form of Alon's combinatorial Nullstellensatz.
This is joint work with Rubén Muñoz Alcázar and Guillermo Vera de Salas from the Universidad Rey Juan Carlos de Madrid.

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Probability and Fixed Divisors of Integer Polynomials

Jean-Luc Chabert

In 1986, Turk stated the following nice formula which gives the 'probability' that the fixed divisor of a polynomial $f \in \mathbb{Z}[X]$ is equal to 1 :

$$\mathrm{Prob}\,\{\, d(f) = 1 \mid f \in \mathbb{Z}[X] \,\} = \prod_{p \in \mathbb{P}} (1 - p^{-p})$$

Recall that the fixed divisor $d(f)$ of f is the gcd of the values of f on $\mathbb{Z}$. By means of Kempner-Bhargava's formula [1], we extend this result to polynomials f with coefficients in the ring $\mathcal{O}_K$ of integers of a number field K and to fixed divisors $d(f, E)$ where $d(f, E)$ denotes the ideal of $\mathcal{O}_K$ generated by the values f on a subset E of $\mathcal{O}_K$:

$$\mathrm{Prob}\,\{\, d(f, E) = \mathcal{O}_K \mid f \in \mathcal{O}_K[X] \,\} = \prod_{\mathfrak{p} \in \mathrm{Max}(\mathcal{O}_K)} \left(1 - \frac{1}{N(\mathfrak{p})^{\nu_{\mathfrak{p}}(E)}}\right)$$

where $\nu_{\mathfrak{p}}(E)$ is equal to the number of classes of E modulo $\mathfrak{p}$.
This will be a talk at the special session "Integer-valued polynomials".

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On Length Densities

Scott Chapman*, Christopher O'Neill, and Vadim Ponomarenko

For a commutative cancellative monoid M, we introduce the notion of the *length density* of both a nonunit $x \in M$, denoted $\mathrm{LD}(x)$, and the entire monoid M, denoted $\mathrm{LD}(M)$. This invariant is related to three widely studied invariants in the theory of non-unit factorizations, $L(x)$, $\ell(x)$, and $\rho(x)$. We consider some general properties of $\mathrm{LD}(x)$ and $\mathrm{LD}(M)$ and give a wide variety of examples using numerical semigroups, Puiseux monoids, and Krull monoids. While we give an example of a monoid M with irrational length density, we show that if M is finitely generated, then $\mathrm{LD}(M)$ is rational and there is a nonunit element $x \in M$ with $\mathrm{LD}(M) = \mathrm{LD}(x)$ (such a monoid is said to have accepted length density). While it is well-known that the much studied asymptotic versions of $L(x)$, $\ell(x)$ and $\rho(x)$ (denoted $\overline{L}(x)$, $\overline{\ell}(x)$, and $\overline{\rho}(x)$) always exist, we show the somewhat surprising result that $\overline{\mathrm{LD}}(x) = \lim_{n\to\infty} \mathrm{LD}(x^n)$ may not exist. We also give some finiteness conditions on M that force the existence of $\overline{\mathrm{LD}}(x)$.

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Idempotent factorizations in premonoids

Laura Cossu

We call premonoid a monoid H with a preorder $\preceq_H$ in its underlying set. An element $a \in H$ is said to be a $\preceq_H$-unit if $a \preceq_H 1_H \preceq_H a$; a $\preceq_H$-irreducible if $a \neq bc$ for all $\preceq_H$-non-units $b, c \in H$ such that $b \prec_H a$ and $c \prec_H a$. It was recently proved in [1] that, if $\preceq_H$ is artinian, every $\preceq_H$-non-unit of H is a finite product of $\preceq_H$-irreducibles. We show that for special choices of H and $\preceq_H$, $\preceq_H$-irreducibles are idempotent. By applying the above factorization theorem or a slight generalization thereof, we recover two classical theorems of J.M. Howie (1966) and J.A. Erdos (1967), as well as a result of J. Fountaine (1991). We also obtain upper bounds on the length of a shortest factorization into $\preceq_H$-irreducibles. This is a joint work with S. Tringali.

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[1] S. Tringali, *An Abstract Factorization Theorem and Some Applications*, e-print (`https://arxiv.org/abs/2102.01598`)

Toric ideals of low dimensional flow polytopes

Mátyás Domokos

Given a lattice polytope one associates with it a toric ideal (an ideal generated by binomials). Flow polytopes form a well studied class of lattice polytopes, and the interest in them is motivated by combinatorial optimization, algebraic statistics, representation theory of Lie groups and quivers. It was proved in [1] that the toric ideal of any flow polytope is generated by elements of degree at most 3. In this talk we shall report about our recent result with Dániel Joó, asserting that the toric ideal of a flow polytope of dimension at most 4 has a quadratic Gröbner basis, with the only exception of the 4-dimensional Birkhoff polytope (consisting of the 3×3 doubly stochastic matrices).

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On Quasihomeomorphisms and Skula Spaces

Othman ECHI

Let $(X, \mathcal{T})$ be a topological space. By the *Skula topology* (or the **b**-topology) on X, we mean the topology $\mathbf{b}(\mathcal{T})$ on X with basis the collection of all $\mathcal{T}$-locally closed sets of X, the resulting space will be denoted by $\mathbf{b}(X)$. We show that the following results hold.

1. $\mathbf{b}(X)$ is an Alexandroff space if and only if the T_0-reflection $\mathrm{T}_0(X)$ of X is a T_D-space.

2. $\mathbf{b}(X)$ is a Noetherian space if and only if $\mathrm{T}_0(X)$ is finite.

3. If we denote by X^* the Alexandroff extension of X, then $\mathbf{b}(X^*) = (\mathbf{b}(X))^*$ if and only if X is a Noetherian quasisober space.

We, also, give an alternative proof of a result due to H. Simmons (2006) concerning the iterated Skula spaces, namely, $\mathbf{b}(\mathbf{b}(\mathbf{b}(X))) = \mathbf{b}(\mathbf{b}(X))$. A space is said to be clopen if its open sets are also closed. In [*Manuscripta Math.* **22** (1977) 365 − 380], Hoffmann has introduced a refinement clopen topology Clop$(\mathcal{T})$ of $\mathcal{T}$: The indiscrete components of $\mathbf{Clop}(X)$ are of the form $C_x = \overline{\{x\}} \cap \mathcal{O}(x)$, where $x \in X$ and $\mathcal{O}(x)$ is the intersection of all open sets of X containing x (equivalently, $C_x = \{y \in X : \overline{\{x\}} = \overline{\{y\}}\}$). We show that $\mathbf{Clop}(X) = \mathbf{b}(\mathbf{b}(X))$. This will be a talk at the special session "Topological methods in ring theory".

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On global defensive k-alliances in zero-divisor graph

Brahim El Alaoui

Let S be a set of vertices of a graph G. Denote by $deg_S(x)$ the degree of a vertex x relative to S. A set of vertices S of a graph G is said to be a defensive k-alliance set in G with k is an integer, if for every $x \in S$, $deg_S(x) \geq deg_{\bar{S}}(x) + k$. The global defensive k-alliance number of G is the minimum cardinality among all dominating defensive k-alliances sets in G.
In this talk, we present our investigation on the global defensive k-alliance number of zero-divisor graphs over finite commutative rings.

This is joint work with Driss Bennis and Khalid Ouarghi.

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Algebraic structures and their spectra

Alberto Facchini

It is possible to attach a 'spectrum' to several algebraic structures: commutative rings with identity, bounded distributives lattices, commutative semirings with identity, commutative C^*-algebras, commutative monoids, abelian ℓ-groups, MV-algebras, continuous lattices, Zariski-Riemann spaces,... In all these cases, the spectrum turns out to be a spectral topological space, i.e., a sober compact space in which the intersection of any two compact open sets is compact, and the compact opens forms a basis for the topology. On some other cases, for instance, for commutative rings without identity or noncommutative rings, one gets a spectrum that is a little less: it is always a sober space, but sometimes compactness is missing, or the intersection of two compact open sets is not necessarily compact. In this talk we will investigate the reason of this 'ubiquity of spectral spaces', giving the proper setting for this kind of questions: properly defined multiplicative lattices. I will present some of the results in [1].
This will be a talk at the special session "Topological methods in ring theory".

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[1] A. Facchini, C. A. Finocchiaro and G. Janelidze, Abstractly constructed prime spectra, submitted for publication, 2021, arXiv:2104.09840.

On the distribution of prime divisors in Krull monoid algebras

Victor Fadinger

In the past, some attempts were made to prove that every class of the class group of a Krull monoid algebra contains a prime divisor. These proofs have open gaps. We will talk about a full proof of this fact. This is joint work with D. Windisch.

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Modules over Trusses vs. Modules over Rings : Internal Direct Sums

Devi Fitri Ferdania

A heap is an algebraic system that can be seen as a group where the existence of its identity element has not been specified. This notion resulting the introduction of a truss as a ring-like system. Thus from modules over rings point of view, modules over trusses were briefly introduced by T. Brzezinski in [1]. The categorical construction on heaps and modules over trusses are considered in [2] and it is shown that the direct sum of two non-empty Abelian heaps is isomorphic to a heap related to the direct sums of the group retracts of both heaps plus $\mathbb{Z}$. Consequently, the internal direct sum of modules over trusses will have some differences to modules over rings. In this research, the definition and some characteristics of the internal direct sum of modules over trusses are constructed and contrasted with the corresponding characteristic on modules over rings.

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[2] Brzezinski, T. and Rybolowicz, B., Modules over Trusses vs. Modules over Rings : Direct Sum and Free Modules, Algebras and Representation Theory, 2020, DOI: 10.1007/s10468-020-10008-8

On quotients of Rees algebras

Carmelo Antonio Finocchiaro

Let R be a commutative ring with 1, let $\mathfrak{a}$ an ideal of R and let $R[\mathfrak{a}T]$ denote the Rees algebra of R with respect to $\mathfrak{a}$. Fix a monic polynomial $f \in R[T]$. The aim of this talk, based on a paper written jointly with M. D'Anna and F. Tartarone, is to introduce and discuss the main properties of the factor ring

$$R_f(\mathfrak{a}) := \frac{R[\mathfrak{a}T]}{fR[T] \cap R[\mathfrak{a}T]}.$$

This ring construction provides a natural generalization of both Nagata's idealizations and amalgamated duplications.

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Cyclotomic exponent sequences of numerical semigroups

Pedro A. García-Sánchez

In this talk we relate numerical semigroups, cyclotomic polynomials and factorizations.

A numerical semigroup is a submonoid under addition of the set of non-negative integers, $\mathbb{N}$, with finite complement in $\mathbb{N}$. Given a numerical semigroup S, its Hilbert series (or generating function) is defined as $\mathrm{H}_S(x) = \sum_{s\in S} x^s \in \mathbb{Z}[\![x]\!]$. Associated to S we set $\mathrm{P}_S(x) = (1 - x)\,\mathrm{H}_S(x)$, which is a polynomial with integer coefficients. It can be shown that there exists unique integers e_j such that $\mathrm{P}_S(x) = \prod_{j=1}^{\infty}(1-x^j)^{e_j}$. We say that $\{e_j\}_{i\in\mathbb{N}\setminus\{0\}}$ is the *cyclotomic exponent sequence* of the numerical semigroup S. We compute its values at the gaps of S, the elements of S with unique representations (factorizations) in terms of minimal generators, and certain Betti elements of B. This allows us to characterize certain semigroup families, and make some progress in proving the conjecture stating that P_S is a product of cyclotomic polynomials if and only if S is a complete intersection [1].

This is a joint work with Alexandru Ciolan, Andrés Herrera-Poyatos, and Pieter Moree [2].

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Implicit difference equation over the ring of polynomials

Anna Goncharuk

Consider the implicit difference linear first order equation over a ring R $bw_{n+1}+f_n = aw_n$, $n = 0, 1, \ldots$, where $a, b, f_n \in R$. This equation may have no solution over this ring. Such equations over $\mathbb{Z}$ was studying using p-adic topology on $\mathbb{Z}$. Let us consider the problem of finding a polynomial solution of this equation, where b, a, f_n are polynomials from $K[z]$ with the coefficients from a field K with characteristic zero. For this we consider the wider ring of formal power series $K[[x]]$, and prove the following theorem.

Theorem. Suppose R is a valuation ring of a complete field with a non-Archimedean valuation $|\cdot|$, $a_i, f_i \in R$. If $|a_j| < |a_0| = 1$, then there is a unique sequence $\{w_n\}$, which satisfies the implicit difference equation $a_m w_{n+m} + a_{m-1} w_{n+m-1} + \ldots + a_1 w_{n+1} + a_0 w_n = f_n$, $n = 0, 1, 2, \ldots$

For the first order equation we have the simple explicit formula for the solution. Let us apply this theorem for $R = K[[x]]$. Now it is enough to check if the solution founded is a polynomial. The described scheme is useful for finding a polynomial solution in specific cases.

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This will be a talk at the special session "Topological methods in ring theory".

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Atomicity and the ascending chain condition on principal ideals

Felix Gotti

Back in 1968, P. Cohn [1] asserted that every atomic integral domain satisfies the ascending chain condition on principal ideals (ACCP). Six years later, A. Grams [2] disproved Cohn's assertion by providing an explicit construction of an atomic domain that does not satisfy the ACCP. Only a few classes of atomic domains without the ACCP have been found so far, even though both the atomic and the ACCP properties have been systematically investigated since then. In this talk, we will discuss generalizations of the construction given by Grams along with further recent progress in this direction.

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Integral domains characterized by a Bezout Property on intersections of principal ideals

Lorenzo Guerrieri

In this talk we study two classes of integral domains. We call an integral domain a Bezout intersection domain (BID) if every finite intersection of its principal ideals is finitely generated only when it is principal. We call it a Strong Bezout intersection domain (SBID) if every finite intersection of principal ideals is always non-finitely generated except in the case of containment of one of the principal ideals in all the others. We relate these classes to other well-studied classes of integral domains, to star operations and to classical and new ring constructions. (Joint work with K. Alan Loper)

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Zariski's finiteness theorem and properties of some ring of invariants

Sudarshan Gurjar

I wish to discuss two theorems which belong to the interface of commutative algebra and invariant theory, both of whose proofs use topology in a crutial way:
One is a criterion for the quotient $\mathbb{A}^3_C//\mathbb{G}_a$, to be a Zariski locally trivial $\mathbb{A}^2$-bundle over C, where C is a smooth complex curve. More precisely we show that if R is a regular complex affine domain of dimension 1 and suppose the additive group $\mathbb{G}_a$ acts by R-automorphisms on $R[X, Y, Z]$ such that the singular fibers of the quotient map Spec $R[X, Y, Z]$ to Spec $R[X, Y, Z]^{\mathbb{G}_a}$ are normal, then this quotient map is a locally trivial $\mathbb{A}^2$-bundle.
The other result being a new proof of a special case of the Zariski finiteness theorem, namely : Let T be an affine factorial domain over $\mathbb{C}$. Let S be an inert subring of T such that the transcendental degree of S over $\mathbb{C}$ is 2. Then S is finitely generated algebra over $\mathbb{C}$.
The novelty feature of the proofs is invoking some algebraic topology.
All these results are obtained in joint work with R.V. Gurjar and B. Hajra and published in Transformation Groups journal.

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On the class group of formal power series rings

Ahmed Hamed

Let $\mathrm{Cl}_t(D)$ denote the t-class group of an integral domain D. P. Samuel ([2]) has established that if D is a Krull domain, then the mapping $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[\![X]\!])$, is injective and if D is a regular UFD, then $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[\![X]\!])$, is bijective. Later, L. Claborn ([1]) extended this result in case D is a regular Noetherian domain. In this paper we prove that the mapping $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[\![X]\!])$; $[I] \mapsto [(I.D[\![X]\!])_t]$ is an injective homomorphism and in case of an integral domain D such that each $v-$invertible $v-$ideal of D has $v-$finite type, we give an equivalent condition for $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[\![X]\!])$, to be bijective, thus generalizing the result of Claborn.
This will be a talk at the special session "Topological methods in ring theory".

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Class (semi)groups and atomicity in Prüfer domains

Richard Erwin Hasenauer

We will explore the relationship between factorization properties of Prüfer domains with their corresponding ideal class groups and ideal class semigroups. We will also construct a generating set for the ideal class semigroup of a sequence domain.

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Enveloping tilting classes over commutative rings: the n-dimensional case

Dolors HERBERA

Given a class $\mathcal{C}$ of modules over a ring R it is a difficult and challenging problem to give intrinsic conditions on the class that ensure that any module has an envelope and/or a cover in the class $\mathcal{C}$, that is, to ensure whether the class $\mathcal{C}$ is enveloping and/or covering.
In the case of a cotorsion pair $C = (\mathcal{A}, \mathcal{B})$ there are plenty of interesting results around this topic. If C is an n-tilting cotorsion pair, that is, if $\mathcal{B}$ is the class of Ext-orthogonal modules to a family of strongly finitely presented modules of projective dimension at most n, then $\mathcal{A}$ is covering if and only if $\mathcal{A}$ is closed under direct limits and, in this case, the n-tilting class $\mathcal{B}$ provides also for envelopes [1]. It is easy to find examples showing that $\mathcal{B}$-enveloping, does not imply that $\mathcal{A}$ is covering.
Bazzoni and Le Gross have recently characteried when a 1-tilting class over a commutative ring R is enveloping [2]. In this talk we will see some results for general n-tilting classes over commutative rings. The basic tool is the characterization of n-tilting cotorsion pairs over commutative rings due to Hrbek and Stovicek [3]. Our results show that, for a ring R, having certain enveloping n-tilting classes is closely related to R being *Cohen-Maculay* but in the setting of general commutative rings.
The talk is based on work in progress with Giovanna Le Gross.

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Families of Integral Domains Inducing Special Set Relations and Set Systems

Federico Infusino

Given integral domain U, we define the collection $\mathcal{X}_U$ of all the *extensible subdomains* of U, which will be characterized through a relation $\leftarrow_{mod}$ between subsets of U and involving finitely generated unitary submodules of U and, equivalently, through a second relation between n-tuples $(u_1, \ldots, u_n)$ of elements of U and subdomains D of U, defined by the vanishing in $(u_1, \ldots, u_n)$ of a specific tipology of polynomials in several variables. Next, we see how to use $\leftarrow_{mod}$ in order to induce a classification of specific families of subdomains, whose main properties will be studied by means of pairs (e, ξ) constituted by an idempotent ring endomorphism ξ having the principal ideal generated by $e \in U^*$ as its kernel.

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On centralizers of generalized matrix algebras

Aisha Jabeen

In this talk, we discuss centralizers on generalized matrix algebras and obtain the necessary and sufficient conditions for a Lie centralizer map to be proper. Further, we discuss that every Jordan centralizer is a centralizer on generalized matrix algebras under certain assumptions.

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The Jacobian conjecture in terms of square-freeness

Piotr Jędrzejewicz

The motivation of the talk is connected with the following properties: all square-free elements / all atoms of a submonoid are square-free in a factorial monoid. The first aim is to explain how these properties are related to the Jacobian conjecture – joint results with J. Zieliński, and results of M. de Bondt and D. Yan. The second aim is to present relations of these properties to closedness with respect to some square-free factorizations – joint work with M. Marciniak, Ł. Matysiak and J. Zieliński.

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Iterated Differential Polynomial Rings and the Behrens Radical

Steven Jin

We first survey the radical properties of differential polynomial rings. Next, we discuss recent work [3] in which we show that a large of class of iterated differential polynomial rings over locally nilpotent rings are Behrens radical. This extends results of Chebotar [1] and Chen, et al [2].

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Generalized power series with a limited number of factorizations

Jason Juett

Roughly speaking, generalized power series rings are the power series analog of semigroup rings. They have proven themselves a useful tool with a wide range of mathematical applications, such as the study of ordered or valued fields, calculus (Taylor series), complex analysis (Laurent series), and Conway's omnific integers. Several past authors have studied questions related to unique factorization of generalized power series. In this talk we examine the broader topic of rings of generalized power series where unique factorization does not necessarily hold but (in a sense we will make precise) the number of distinct factorizations of elements is at least limited. Special cases of our general results include new results about "limited factorization" in (Laurent) power series rings, (Laurent) polynomial rings, and the "large polynomial rings" of Halter-Koch. Along the way to our main results, we study Krull domains and Cohen-Kaplansky rings of generalized power series. This talk is based on research done by the presenter in collaboration with Ngoc Aylesworth (Texas State University) [1].

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On the arithmetic of monoids of ideals

Azeem Khadam

In this talk we present our recent work (joint with Alfred Geroldinger) on the arithmetic of monoid of ideals. Our focus will be on the following two monoids:

(i) monoid of invertible ideals of a weakly Krull domain, and

(ii) monoid of nonzero ideals of a polynomial domain.

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On power invariant rings

Mohamed Khalifa

O'Malley called a ring R to be power invariant (respectively strongly power invariant) if whenever S is a ring such that $R[[X]]$ is isomorphic to $S[[X]]$ (respectively whenever S is a ring and φ is an isomorphism of $R[[X]]$ onto $S[[X]]$), then R and S are isomorphic (respectively then there exists an S-automorphism ψ of $S[[X]]$ such that $\psi(X) = \varphi(X)$). We prove that a ring R is power invariant in each of the following cases (1) R is a domain in which $\mathrm{Jac}(R)$ (i.e., the Jacobson radical of R) is comparable to each radical ideal of R and (2) R is a Prufer domain. Also in each of the aforementioned case, we prove that either R is strongly power invariant or R is isomorphic to a quasi-local power series ring.
This will be a talk at the special session "Integer-valued polynomials".

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Morphic elements in regular near-rings

Alex Samuel Bamunoba, Ivan Philly Kimuli* and David Ssevviiri

We define morphic near-ring elements and study their behavior in regular near-rings. We show that the class of left morphic regular near-rings is properly contained between the classes of left strongly regular and unit-regular near-rings.

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Representation type of commutative Noetherian rings

Lee Klingler

In joint work with L. Levy, we defined a class of commutative Noetherian rings which we called *Dedekind-like* and gave a careful construction of all finitely generated modules over such rings. Inspired by the tame-wild dichotomy theorem for finite dimensional algebras over algebraically closed fields, we proved that every indecomposable commutative Noetherian ring is either a homomorphic image of a Dedekind-like ring or of wild representation type or one of a small class of local Artinian rings of composition length four.
In this talk, I shall define Dedekind-like rings, giving a brief description of their finitely generated modules, and explain the notion of tame versus wild representation type. After giving the precise statements of our theorems, I shall discuss the extent to which they are in fact dichotomy theorems. Relevant to this question is recent joint work with R. Wiegand and S. Wiegand, in which we attempt to show that Dedekind-like rings cannot be wild.

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Subrings of polynomial rings and divisibility

Miroslav Korbelář

A finitely generated commutative ring R (i.e., a factor of a polynomial ring $T = \mathbb{Z}[x_1, \ldots, x_n]$) has to be trivial, provided that its additive group is divisible. Similarly, in such a ring R the additive group has to be torsion if this group is divisible with respect to an infite set of prime numbers. We show that the latter implication remains true also for rings that are factors of those subrings S of T, such that S is generated by some set of monomials. A similar property can also be extended to commutative semirings.

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Sums of n-th powers in rings of formal power series

Tomasz Kowalczyk

Let n be a positive integer. For a (commutative) ring A we define its nth Pythagoras number $p_n(A)$ as the smallest positive integer g such that any sum of nth powers in A can be expressed as a sum of at most g nth powers in A. If such number doest not exist, we put $p_n(A) = \infty$. Theory of Pythagoras numbers has a rich literature which is mainly devoted to study the case $n = 2$. During the talk, I will make a short introduction about higher Pythagoras numbers in various contexts. Let K be a field. I will show how to compute $p_n(K[[x]])$ and $p_n(K((x)))$, where x is a single variable, provided that we know the value of $p_n(K)$ and the nth level of K (under some mild assumptions on K and n). Based on a joint work with Piotr Miska.

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The locally nilradical for modules over commutative rings

Annet KYOMUHANGI*, David SSEVVIIRI

Let R be a commutative unital ring and $a \in R$. We introduce and study properties of a functor $a\Gamma_a(-)$, called the locally nilradical on the category of R-modules. $a\Gamma_a(-)$ is a generalisation of both the torsion functor (also called section functor) and Baer's lower nilradical for modules. Several local-global properties of the functor $a\Gamma_a(-)$ are established. As an application, results about reduced R-modules are obtained and hitherto unknown ring theoretic radicals as well as structural theorems are deduced.

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The i-extended zero-divisor graphs of commutative rings

Raja L'Hamri

The zero-divisor graphs of commutative rings have been used to build bridges between ring theory and graph theory. Namely, they have been used to characterize many ring properties in terms of graphic ones. However, many results are established only for reduced rings because a zero-divisor graph defined in the classical manner lacks the information on relationship between powers of zero-divisors. The aim of this paper is to remedy this situation by introducing a parametrized family of graphs $\{\overline{\Gamma}_i(R)\}_{i\in\mathbb{N}^*}$, for a ring R, which reveals more of the relationship between powers of zero-divisors as follows: For each $i \in \mathbb{N}^*$, $\overline{\Gamma}_i(R)$ is the simple graph whose vertex set is the set of non-zero zero-divisors such that two distinct vertices x and y are joined by an edge if there exist two positive integers $n \leq i$ and $m \leq i$ such that $x^n y^m = 0$ with $x^n \neq 0$ and $y^m \neq 0$. Our aim is to study in detail the behaviour of the filtration $\{\overline{\Gamma}_i(R)\}_{i\in\mathbb{N}^*}$ as well as the relations between its terms. We give answers to several interesting and natural questions that arise in this context. In particular, we characterize girth and diameter of $\overline{\Gamma}_i(R)$ and give various examples.

This is joint work with Driss Bennis, Brahim El Alaoui, Brahim Fahid and Michał Farnik.

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Number of Solutions to $x^2 + y^2 + z^2 = 0$ in $\mathbb{Z}_p$

Aihua Li*, Xiao Wang

Consider an odd prime number $p \equiv 2 \pmod 3$. In this paper, the number of solutions to the Diophantine equation $x^2 + y^2 + z^2$ in $\mathbb{Z}_p$ is calculated. The focus is on counting the number of solution triples (a, b, c), where $a, b, c \in \mathbb{Z}_p \setminus \{0\}$. These triples can produce solutions to the Diophantine equation equation $x^m + y^m + z^m = 0$ where m is any integer in the form of $m = 2 \cdot 3^k$ with k being a non-negative integer. We classify the solution triples into two types: those with a, b, c all distinct or exactly two are the same. Enumeration of each type is given.
This will be a talk at the special session "Integer-valued polynomials".

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A characterization of Rédei permutations with 1- and p-cyles

Ariane Masuda

Let $\mathbb{F}_q$ be a finite field of odd characteristic. We characterize Rédei functions that induce permutations over $\mathbb{P}^1(\mathbb{F}_q)$ with 1- and p-cycles for a prime p. As a consequence, we obtain a formula for their total number. For an odd p, we show that there exists a Rédei permutation over $\mathbb{P}^1(\mathbb{F}_q)$ with 1- and p-cycles if and only if $q-1$ or $q+1$ has a prime factor of the form $pk+1$ or is divisible by p^2. We also present explicit formulas for Rédei involutions based on the number of fixed points, and procedures to construct Rédei permutations with a prescribed number of fixed points and j-cycles for $j \in \{3,4,5\}$. This is joint work with Juliane Capaverde and Virgínia Rodrigues.

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z-ideals and higher order z-ideals in commutative rings

Ahmed Maatallah

This talk deals with special ideals of a commutative ring namely the z-ideals. This notion was first introduced by "Kohls" in 1957 during the study of the ring of real-valued continuous functions on a Tychonoff space X, $C(X)$. For a ring R, we give a relation between the z-ideals of R and those of the formal power series rings in an infinite set of indeterminates over R. Consider $R[[X_\Lambda]]_3$ and its subrings $R[[X_\Lambda]]_1$, $R[[X_\Lambda]]_2$, $R[[X_\Lambda]]_\alpha$, where α is an infinite cardinal number. In fact, a z-ideal of one of the rings defined above is of the form $I + (X_\Lambda)_i$ where $i = 1, 2, 3$ or an infinite cardinal number and I is a z-ideal of R. Finally, we will study a generalization of this concept that was introduced by T. Dube and O. Ighedo, namely, higher order z-ideal in commutative rings, in 2016.

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p-adic denseness of fractions of values of polynomials with integer coefficients

Piotr Miska

For any set $A \subset \mathbb{Z}$ we define its ratio set as

$$R(A) = \{a/b : a, b \in A, b \neq 0\}.$$

The topic of denseness of sets $R(A)$ in positive real half-line generated a lot of literature. Recently, the subject of denseness of these sets in p-adic fieds is also under investigation and, as a new area of study, has many open problems. The talk is devoted to one of them.
Let $f \in \mathbb{Z}[X]$ be fixed. Then, one can ask how large is the set

$$\mathbb{P}_f = \{p \in \mathbb{P} : R(f(\mathbb{Z})) \text{ is dense in } \mathbb{Q}_p\}.$$

During the talk we will show that if f is irreducible over $\mathbb{Z}$, then the set $\mathbb{P}_f$ has the asymptotic density $d_{\mathbb{P}}(\mathbb{P}_f)$ with respect to the set of prime numbers and this density is positive. Moreover, we will focus on the set of al the possible values of $d_{\mathbb{P}}(\mathbb{P}_f)$, when $f \in \mathbb{Z}[X]$ runs over all the irreducible polynomials over $\mathbb{Z}$.

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Notions of Unique Factorization in Commutative Rings with Zero Divisors

Christopher Park Mooney

In this talk, we give an overview of a recent paper, [1], which analyzes many ways in which unique factorization domains can be generalized to rings with zero-divisors. Numerous authors have extended this notion in various ways throughout the literature, so this article seeks to determine which notions are equivalent and determine the relationship between these various definitions. In most cases, we are able to provide a structural description of these different classes of unique factorization rings. We present these definitions of unique factorization rings, demonstrate the diagrams which explain the relationships between these possibilities, as well as (if time permits) give some samples of the structural descriptions of some of these classes of rings.

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On the Structure of Finitely Generated Modules

Khaerudin SALEH, Pudji ASTUTI, Intan MUCHTADI-ALAMSYAH*

Given a finitely generated module over a principle ideal domain, we will present the structure of its endomorphism ring and identification of a fully invariant submodule in term of a cyclic decomposition of the module. Furthermore we will use this identification to characterize S-prime submodules.

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Integer-valued polynomials over upper triangular matrix rings

Ali Reza Naghipour

S. Frisch showed that the set of integer-valued polynomials over triangular matrix ring $\mathrm{Int}_{T_n(K)}(T_n(D)) := \{f \in T_n(K) | f(T_n(D)) \subseteq f(T_n(D))\}$ is a ring, where D is an integral domain with quotient field K [1]. Let $R_1 \subseteq R_2$ be commutative rings with identity. In this talk, we study the set $\mathrm{Int}_{T_n(R_2)}(W, T_n(R_1)) := \{f \in T_n(R_2) | f(W) \subseteq f(T_n(D))\}$ for some subsets $W \subseteq T_n(R_1)$. We generalize Frisch's result and show that $\mathrm{Int}_{T_n(R_2)}(T_n(R_1)) := \mathrm{Int}_{T_n(R_2)}(T_n(R_1), T_n(R_1))$ is a ring. Finally, we state a lower bound of Krull dimension of the integer-valued polynomials over upper triangular matrix.
This will be a talk at the special session "Integer-valued polynomials".

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A graph-theoretic criterion for absolute irreducibility in rings of integer-valued polynomials

Sarah Nakato

An irreducible element of a commutative ring is called *absolutely irreducible* if none of its powers has more than one (essentially different) factorizations into irreducibles; otherwise, it is called *non-absolutely irreducible*. The non-absolutely irreducible elements have been used in several contexts, for instance, to characterize number fields with certain class groups. In the context of non-unique factorizations into irreducibles, non-absolutely irreducible elements are important in investigating patterns of factorizations. In this talk, we discuss absolutely and non-absolutely irreducible elements in the ring

$$\mathrm{Int}(D) = \{f \in K[x] \mid f(D) \in D\}$$

of integer-valued polynomials on a principal ideal domain D with quotient field K. We discus a graph-theoretic sufficient condition for a polynomial $f \in \mathrm{Int}(D)$ to be absolutely irreducible. Furthermore, we show that our criterion is necessary and sufficient in the special case of polynomials with square-free denominator. This is joint work with Sophie Frisch.
This will be a talk at the special session "Integer-valued polynomials".

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[1] Sophie Frisch and Sarah Nakato, A graph-theoretic criterion for absolute irreducibility of integer-valued polynomials with square-free denominator, Communications in Algebra, 2020, 3716–3723, 48

Strong tilting of algebras and its iteration

Zahra Nazemian

One important consequence of the existence of a strong tilting module over a finite dimensional algebra is its connection to the finitistic dimension conjecture. That is, if Λ is a finite dimensional algebra and ${}_{\Lambda}T$ is a strong tilting Λ-module, then the left finitistic dimension of Λ is finite and equal to the projective dimension of T. In this talk, by a constructive method, we show that the existence of strong tilting module over Λ depends to the existence of such module over corner algebra $e\Lambda e$, where e is a particular idempotent of Λ. We also show that the iteration process becomes periodic for any finite dimensional algebra that allows infinite strong iteration. This talk is based on joint ongoing work with Manuel Saorín and Birge Huisgen-Zimmermann.

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On Krull Modules and v-operations in Modules

Mu'amar Musa Nurwigantara*,
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Hidetoshi Marubayashi,
Sri Wahyuni

Krull module as a generalization of Krull domain have attracted some researchers, especially to see the relationship with Dedekind modules. While paper [1] studies Krull modules in case of faithful multiplication modules, paper [2] studies them in case of finitely generated torsion-free modules. In characterizing Krull modules, these two papers used different v-operation, which sparks two different views of Krull modules. In this talk, we want to see the relationship between those two v-operations, as well as between those two characterization of Krull modules.

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On product-one sequences over dihedral groups

Jun Seok Oh

Let G be a finite group. A sequence over G means a finite sequence of terms from G, where repetition is allowed and the order is disregarded. A product-one sequence is a sequence whose elements can be ordered such that their product equals the identity element of the group. The set of all product-one sequences over G (with concatenation of sequences as the operation) is a finitely generated C-monoid. Product-one sequences over dihedral groups have a variety of extremal properties. In this talk, we provide a detailed investigation, with methods from arithmetic combinatorics, of the arithmetic of the monoid of product-one sequences over dihedral groups. Joint work with A. Geroldinger, D. J. Grynkiewicz, and Q. Zhong.

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Quadratic transforms and intersections of valuation rings

Bruce Olberding

This talk focuses on the integrally closed rings between a two-dimensional regular local ring D and its quotient field. The set of all such rings that are themselves two-dimensional regular local rings forms a tree ordered by inclusion. Each ring in this tree, the *quadratic tree* of D, can be obtained by a sequence of iterated quadratic transforms, a construction that has a natural interpretation in algebraic geometry involving the blow up at a maximal ideal. The valuation overrings that dominate D can also be interpreted using this tree.
The quadratic tree admits a topology that is useful in describing the intersections of the regular local rings in the tree. The quadratic tree and the intersections of its rings are important for understanding blow ups of Spec(D), desingularizing projective models over D, and describing valuation overrings of D. We explain these applications and examine some of the connections between the topology of the tree and the nature of the intersections of rings from the tree, as well as the intersections of certain sets of valuation rings associated to the tree. The talk is based on joint work with William Heinzer and Alan Loper.

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A characterization of large Dedekind domains

Greg Oman

An extremely elementary number-theoretic observation is that every nonzero integer has but finitely many integer divisors. It follows immediately that every nonzero ideal of the ring $\mathbb{Z}$ of integers is contained in but finitely many ideals of $\mathbb{Z}$. Let us agree to call a commutative domain D *ideal upper finite* if every nonzero ideal of D is contained in by finitely many ideals of D. In this talk, we establish that a domain D of cardinality strictly greater than $2^{\aleph_0}$ is ideal upper finite iff D is a Dedekind domain. However, if κ is a cardinal such that $\aleph_0 \leq \kappa \leq 2^{\aleph_0}$, then there is an ideal upper finite domain D of cardinality κ which is not Dedekind (this can be shown in ZFC).

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Distribution of factorization lengths of large numerical semigroup elements

Christopher O'Neill

A numerical semigroup S is a subset of the nonnegative integers that is closed under addition, and a factorization of $n \in S$ is an expression of n as a sum of generators of S. Many factorization-theoretic invariants involve factorization lengths, but most concern extremal lengths (e.g. maximum and minimum factorization lengths). In this talk, we examine several factorization invariants (e.g. mean, median, and mode lengths) that depend on "medium-length factorizations". No familiarity with numerical semigroups or factorization theory will be assumed for this talk.

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Ostrowski's Fundamentalsatz, pseudo-convergent sequences and integer-valued rational functions

Giulio Peruginelli

In this talk we will review the Ostrowski's notion of pseudo-convergent sequence in a rank one valued field (K, v) that he introduced in 1935 in order to describe all the possible extensions of v to the field of rational functions $K(X)$.

We will emphasize how this notion has been recently employed in the context of rings of integer-valued polynomials by some authors (Chabert, Loper, Werner, etc...), along with the notion of pseudo-monotone sequence introduced by Chabert in 2010. The results of these authors finally led to a criterion which establishes for which subsets S of V the ring $\mathrm{Int}(S, V) = \{f \in K[X] \mid f(S) \subseteq V\}$ is Prüfer domain.

We will also overview some recent work joint with Dario Spirito about valuation domains of $K(X)$ associated to pseudo-monotone sequences, without any restriction on the rank of V.

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Exponential Puiseux Semirings

Harold Polo

Exponential Puiseux semirings are additive submonoids of $\mathbb{Q}_{\geq 0}$ generated by almost all of the nonnegative powers of a positive rational number, and they are natural generalizations of rational cyclic semirings. In this talk, we will give an overview of some of the atomic properties and factorization invariants of this class of semirings.

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Ring of polynomials over $*-$rings

Puguh Wahyu Prasetyo*
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A semiprime ring A is called a $*-$ring if $\beta(R/I) = R/I$ for any nonzero ideal I of A, where β is the prime radical. In this paper, we show that every rings of polynomials over $*-$ring is also a $*-$ring.

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Lattices, Spectral Spaces, and Closure Operations on Idempotent Semirings

Samarpita Ray

Spectral spaces, introduced by Hochster in [1], are topological spaces homeomorphic to the prime spectra of commutative rings. In this talk, we will discuss connections between spectral spaces and idempotent semirings, which are algebraic structures receiving a lot of attention due to its several applications to tropical geometry. We first prove that a space is spectral if and only if it is the *saturated prime spectrum* of an idempotent semiring. In fact, we enrich Hochster's theorem by constructing a subcategory of idempotent semirings which is equivalent to the category of spectral spaces. We further provide examples of spectral spaces arising from sets of congruence relations of semirings. In particular, we prove that the space of valuations and the space of prime congruences on an idempotent semiring are spectral, and there is a natural bijection of sets between the two; this shows a stark difference between rings and idempotent semirings. If time permits, we will then present certain aspects of commutative algebra of idempotent semirings that we develop in the process, like closure operations. This is a joint work with Jaiung Jun and Jeffrey Tolliver.
This will be a talk at the special session "Topological methods in ring theory".

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On rings with one-absorbing factorization

Andreas Reinhart

This talk is based on a joint work with Abdelhaq El Khalfi, Mohammed Issoual and Najib Mahdou. Let R be a commutative ring with identity and let I be a proper ideal of R. We say that I is an OA-ideal of R if for all nonunits $x, y, z \in R$ with $xyz \in I$, it follows that $xy \in I$ or $z \in I$. Furthermore, I is called a 2-absorbing ideal of R if for all $x, y, z \in R$ with $xyz \in I$, we have that $xy \in I$ or $xz \in I$ or $yz \in I$. Clearly, if I is a prime ideal, then I is an OA-ideal and if I is an OA-ideal, then I is a 2-absorbing ideal. A finite product of OA-ideals of R which is equal to I is called an OA-factorization of I. We say that R is an OAF-ring if every proper ideal of R has an OA-factorization. In this talk we present characterizations of OAF-rings and rings whose proper principal ideals (resp. proper 2-generated ideals) have an OA-factorization. We provide nontrivial examples of OAF-rings and discuss several ring theoretic constructions (with special emphasis on trivial ring extensions) with regard to OA-factorizations.

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On extremal product-one free sequences over $C_n \rtimes_s C_2$ and the Gao conjecture for $D_{2n} \times C_2$

Sávio Ribas

Let G be a finite multiplicative group. The Gao and Gao-Zhuang conjectures assert that $\mathsf{s}(G) = \eta(G) + \exp(G) - 1$ and $\mathsf{E}(G) = \mathsf{d}(G) + |G|$, respectively, where $\mathsf{E}(G)$, $\mathsf{d}(G)$, $\mathsf{s}(G)$ and $\eta(G)$ denote the Gao constant, small Davenport constant, Erdős-Ginzburg-Ziv constant and η constant of G. Let C_n be the cyclic group of order n and let D_{2n} be the dihedral group of order $2n$. In this talk, we classify all the product-one free sequences over $C_n \rtimes_s C_2$ of length $\mathsf{d}(C_n \rtimes_s C_2)$ and prove both conjectures over $D_{2n} \rtimes C_2$. This is a joint work with D.V. Avelar, F.E. Brochero Martínez, A. Lemos and B.K. Moriya.

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Completely split absolutely irreducible integer-valued polynomials over discrete valuation domains

Roswitha Rissner

Non-unique factorization of elements into irreducibles has been observed in the ring of integer-valued polynomials and its generalizations. It is known that every (multi-)set consisting of integers greater than 1 can be realized as the (multi-)set of lengths of an integer-valued polynomial over a Dedekind domain D with infinitely many maximal ideals whose residue fields are finite. Moreover, under the same assumptions on D, $\mathrm{Int}(D)$ is not transfer Krull. The proofs of these two statements are build on the constructions of integer-valued polynomial whose factorization behaviour can be fully controlled. For this, it is crucial to avoid the situation of a factorization in which an irreducible factor occurs more than once. This is because in non-unique factorization domains there is in general no saying how the powers of an irreducible element factor. From a factorization theoretic point of view, one wants therefore identify those elements among the irreducibles whose powers factor uniquely. We call an irreducible element f *absolutely irreducible* or *strong atom* if every power f^n of f has essentially one factorization, namely $f \cdot f \cdots f$ (n times). Irreducible elements which are not absolutely irreducible are known to exist in rings of integer-valued polynomials. In a recent project we were able to characterize the absolutely irreducible polynomials among the completely split polynomials in terms of their root set in $\mathrm{Int}(R)$ where R is a discrete valuation domain with finite residue field.
This is joint work with S. Frisch and S. Nakato.
This will be a talk at the special session "Integer-valued polynomials".

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Differential identities and polynomial growth of their codimensions

Carla Rizzo

Let A be an associative algebra over a field F of characteristic zero. If L is a Lie algebra over F acting on A by derivations, then such an action can be naturally extended to the action of its universal enveloping algebra $U(L)$. In this case we refer to A as algebra with derivations or L-algebra. With these ingredients at hand one studies the polynomials in non commuting variables $x^h = h(x)$, where $h \in U(L)$, vanishing in A, that is the differential identities of A.
One associates to the T_L-ideal $\mathrm{Id}^L(A)$ of all differential identities of A, in a natural way, a numerical sequence $c_n^L(A)$, $n = 1, 2, \ldots$, called the sequence of differential codimensions of A which is the main tool for the quantitative investigation of the polynomial identities of the algebra A. Such a sequence, in case A is finite dimensional satisfying a non trivial identity, is exponentially bounded.
The purpose of this talk is to survey some recent results on the growth of the differential codimensions. Based on the existence of the differential exponent of an L-algebra

$$\exp^L(A) = \lim_{n\to\infty} \sqrt[n]{c_n^L(A)}$$

we shall answer to the question: can one characterize the T_L-ideal of differential identities of polynomial growth?

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On Zaks domains

Moshe ROITMAN

A domain R will be called a *Zaks domain*, if there exists a factorial domain D such that each irreducible element of R remains irreducible in D. In an article of 1980, Abraham Zaks remarked that such a domain is half-factorial, and asked whether the converse is true. I will present a simple counterexample. Moreover, in contrast to half-factorial Krull (or even Dedekind) domains, a Zaks Krull domain with torsion class group is necessarily factorial. Also, a Zaks proper polynomial extension of a Krull domain is factorial. I will characterize Zaks domains, and survey further results, for example::

- There exists an atomic Zaks domain that is not Mori. In particular, a half-factorial domain is not necessarily Mori, thus answering a question of Alfred Geroldinger.

- If some proper polynomial extension of a domain R is half-factorial, then all polynomial extensions (even infinite) of R are half-factorial. This answers in the positive a question of Jim Coykendall: if $R[X]$ is half-factorial, then $R[X, Y]$ is half-factorial.

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Factorization of Polynomials over Quaternion Algebras

Daniel F. Scharler

We discuss results on factorizations into linear factors for (left) polynomials over the real algebra of Hamiltonian quaternions, dual quaternions and split quaternions. Moreover, we present algorithms to compute such factorizations. Polynomials which do not admit a factorization into linear factors can, however, be factored after multiplication by a proper real polynomial (an element of the center). The presented results are also relevant to Euclidean and hyperbolic kinematics as dual quaternions and split quaternions provide parametrizations for the groups of Euclidean and hyperbolic displacements, respectively.

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Generalizations of monoids of zero-sum sequences and applications

Wolfgang Schmid

The arithmetic of monoids of zero-sum sequences over finite abelian groups is a subject of intense study since several decades. They play a central role in understanding the arithmetic of (transfer) Krull monoids with finite class group.

For a finite abelian group $(G, +)$, a sequence $g_1 \dots g_k$ over G is called a zero-sum sequence if $g_1 + \cdots + g_k = 0$ (we consider sequences that just differ by the ordering of the terms as equal). The set of zero-sum sequences over G form monoids with concatenation as operation.

First, we discuss generalization of this concept, including monoids of weighted zero-sum sequences and monoids of subsets of finite abelian groups. Then, some algebraic and arithmetic results on these structures are presented. We conclude with an application to factorizations of norms of algebraic integers.

This is joint work with Safia Boukheche, Kamil Merito and Oscar Ordaz.

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Integer-valued polynomials over matrix rings of number fields

Javad Sedighi Hafshejani

In this talk, we study the ring of integer valued polynomials $\mathrm{Int}(M_n(\mathcal{O}_K)) := \{f \in M_n(K)[x] \mid f(M_n(\mathcal{O}_K)) \subseteq M_n(\mathcal{O}_K)\}$, where K is a number field and $\mathcal{O}_K$ is the ring of algebraic integers of K. We show that for a prime number $p \in \mathbb{Z}$, the polynomial $f_{p,n}(x) := \frac{(x^{p^n}-x)(x^{p^{n-1}}-x)\ldots(x^p-x)}{p}$ is an element of $\mathrm{Int}(M_n(\mathcal{O}_K))$ if and only if p is a totally split prime in $\mathcal{O}_K$. Also, we consider $\mathrm{Int}_{M_n(\mathbb{Q})}(M_n(\mathcal{O}_K)) := \mathrm{Int}(M_n(\mathcal{O}_K)) \bigcap M_n(\mathbb{Q})[x]$. Then, we characterize finite Galois extensions K of $\mathbb{Q}$ in terms of the ring $\mathrm{Int}_{M_n(\mathbb{Q})}(M_n(\mathcal{O}_K))$. In fact, we prove that $\mathrm{Int}_{M_n(\mathbb{Q})}(M_n(\mathcal{O}_K)) = \mathrm{Int}_{M_n(\mathbb{Q})}(M_n(\mathcal{O}_{K'}))$ if and only if $K = K'$, where K, K' are two finite Galois extensions of $\mathbb{Q}$. Finally, we present some results on Noetherian property of the rings $\mathrm{Int}_{M_n(\mathbb{Q})}(M_n(\mathcal{O}_K))$. Then, we obtain many non-Noetherian integral domains, $\mathrm{Int}_{\mathbb{Q}}(\mathcal{O}_K)$, between the ring $\mathbb{Z}[x]$ and the classical ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$ [1].
This will be a talk at the special session "Integer-valued polynomials".

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Completion of unimodular rows over finitely generated rings

Sampat Sharma

R.G. Swan and J. Towber showed that if (a^2, b, c) is a unimodular row over any commutative ring R then it can be completed to an invertible matrix over R. This was strikingly generalised by A.A. Suslin who showed that if $(a_0^{r!}, a_1, \ldots, a_r)$ is a unimodular row over R then it can be completed to an invertible matrix. As a consequence A.A. Suslin proceeds to conclude that if $\frac{1}{r!} \in R$, then a unimodular row $v(X) \in Um_{r+1}(R[X])$ of degree one, with $v(0) = (1, 0, \ldots, 0)$, is completable to an invertible matrix. Then he asked
$(\mathbf{S_r}(\mathbf{R}))$: Let R be a local ring such that $r! \in GL_1(R)$, and let $p = (f_0(X), \ldots, f_r(X)) \in Um_{r+1}(R[X])$ with $p(0) = e_1(= (1, 0, \ldots, 0))$. Is it possible to embed the row p in an invertible matrix?
Due to Suslin, one knows answer to this question when $r = d + 1$, without the assumption $r! \in GL_1(R)$. In 1988, Ravi Rao answered this question in the case when $r = d$.
In this talk we will discuss about the Suslin's question for polynomial extension of finitely generated rings over $\mathbb{Z}$

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On noncommutative bounded factorization domains

Daniel SMERTNIG

A domain R is a bounded factorization (BF) domain if for every $0 \neq a \in R$, there exists $\lambda(a) \in \mathbb{N}_0$ such every factorization of a into atoms has length at most $\lambda(a)$. It is well-known that commutative noetherian domains (and more generally, Mori domains) are BF-domains.
The situation for noncommutative noetherian domains and prime rings is less clear. We present some recent results in this direction, in particular sufficient conditions for such rings to be BF.
This is joint work with Jason Bell.

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Some open questions on finite nilpotent rings related to the quantum Yang-Baxter equation

Agata Smoktunowicz

In around 2005, Wolfgang Rump introduced braces, a generalisation of nilpotent rings to describe all involutive, non-degenerate set theoretic solutions of the Yang-Baxter equation. This formulation then rapidly found application in other research areas.
Definition. A set A with binary operations of addition $+$, and multiplication $\circ$ is a brace if $(A;+)$ is an abelian group, $(A;\circ)$ is a group and $a\circ(b+c)+a = a\circ b + a\circ c$ for every $a, b, c \in A$. It follows from this definition that every nilpotent ring with the usual addition and with multiplication $a\circ b = ab+a+b$ is a brace.
In algebraic number theory there is a correspondence between braces and Hopf-Galois extensions of abelian type first observed by David Bachiller. We will mention some open problems on commutative nilpotent rings which appear in this situation.
On the other hand, Anastasia Doikou, Aryan Ghobadi and Robert Weston have recently discovered connections between braces and quantum integrable systems. In particular, to find solutions of the set-theoretic reflection equation it is needed to solve problems on identities in nilpotent rings. Because previously the theory of functional identities was mainly developed for prime rings, and for the reflection equation we only consider nilpotent rings, there are no known methods for solving such problems. We will mention some open questions related to commutative nilpotent rings which appear in this situation.

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Jaffard and pre-Jaffard families

Dario Spirito

Jaffard families are families of overrings that allow to generalize the concept of h-local domains and extend several results on factorization properties. In this talk, I will introduce the concept of *pre-Jaffard family*, a further generalization of the concept of Jaffard families obtained trading the local finiteness of the family for compactness in the Zariski topology. Then, I will construct from a pre-Jaffard family Θ a sequence of overrings (the *derived sequence* of Θ) reminiscent of the derived family of a topological space, and show how it allows to prove factorization properties of stable semistar operations and singular length functions in this more general context. Finally, I will associate to a pre-Jaffard family an ordinal number which generalizes the notions of stable and dull degree of a Prüfer domain.
This will be a talk at the special session "Topological methods in ring theory".

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Equidistribution in number fields

Anna Szumowicz

The notion of simultaneous $\mathfrak{p}$-orderings was introduced by Bhargava in his work on integer-valued polynomials. Let k be a number field and let $\mathcal{O}_k$ be its rings of integers. A sequence of elements in $\mathcal{O}_k$ is a simultaneous $\mathfrak{p}$-ordering if it is equidistributed modulo every power of every prime ideal in $\mathcal{O}_k$. We show that the only number field for which $\mathcal{O}_k$ admits a simultaneous $\mathfrak{p}$-ordering is $\mathbb{Q}$. Based on a joint work with Mikolaj Fraczyk.

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Pólya groups of some real bicyclic biquadratic number fields

Mohammed Taous

Let $\mathcal{O}_K$ and C_K be respectively the ring of integers and the class group of a number field K. For each integer $q \geq 2$, denote by $\prod_q(K)$ the product of all the maximal ideals of $\mathcal{O}_K$ with norm q, if these ideals do not exist we set $\prod_q(K) = \mathcal{O}_K$. The Pólya group of K is the subgroup of C_K generated by the classes of the ideals $\prod_q(K)$, and K is called a Pólya field if the module of integer-valued polynomials over $\mathcal{O}_K$ has a regular basis. In this work, we determine Pólya group of some real bicyclic biquadratic number fields.
This will be a talk at the special session "Integer-valued polynomials".

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PvMD properties of Integer-valued polynomial rings and construction of examples

Francesca Tartarone

I will go through some results about characterization of Prüfer v-multiplication domains obtained in collaboration with Paul-Jean Cahen and show intereactions with topological properties of PvMD's thorugh the construction of Examples of integer-valued polynomial rings. This will be a talk at the special session "Integer-valued polynomials".

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Maximal subgroups of branch groups

Anitha Thillaisundaram

Stemming from the Burnside Problem, branch groups are an interest in their own right. We look at questions concerning branch groups and their maximal subgroups. In particular we mention a connection to a conjecture by Kaplansky on the group algebra $F[G]$ of a finitely generated group G over a field F of characteristic p.

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Some Little Steps towards a Grand Theory of Factorization

Salvatore Tringali

Let H be a monoid (written multiplicatively) with identity 1_H and $\preceq$ be a preorder (i.e., a reflexive and transitive binary relation) on H. (Note that H need not be commutative or cancellative.)
We take an element $u \in H$ to be a $\preceq$-unit if $1_H \preceq u \preceq 1_H$, or else we call u a $\preceq$-non-unit. Accordingly, we refer to a $\preceq$-non-unit $a \in H$ as a $\preceq$-irreducible if $a \neq xy$ for all $\preceq$-non-units $x, y \in H$ with $x \prec a$ and $y \prec a$.
On the other hand, we say that $\preceq$ is an artinian preorder if there is no infinite sequence $x_1, x_2, \ldots$ of elements of H with $x_{i+1} \prec x_i$ for each i, where $y \prec z$ means that $y \preceq z$ and $z \not\preceq y$.
It was recently proved in [1] that, if $\preceq$ is an artinian preorder, then every $\preceq$-non-unit of H factors into a (non-empty, finite) product of $\preceq$-irreducibles. Due to its very generality, the result has a rather simple proof and a wide range of applications, some of which will be the object of this talk. Further applications (to idempotent factorizations of singular elements in various classes of rings) will be discussed by L. Cossu in a subsequent talk.

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[1] S. Tringali, *An Abstract Factorization Theorem and Some Applications*, e-print (https://arxiv.org/abs/2102.01598)

Complete quasi-one-fibered ideals in dimension two

Veronique Van Lierde

We use the theory of degree functions developed by D. Rees and R.Y. Sharp as well as some results about 2-dimensional regular local rings to study complete quasi-one-fibered ideals in a 2-dimensional Muhly local domain. The 2-dimensional Muhly rational singularities form a class of local rings that contains the 2-dimensional regular local rings. The theory of complete ideals in 2-dimensional regular local rings started with Zariski and was extended by several others.
Let (R, m) be a 2-dimensional rational singularity. (R, m) is called a Muhly domain if (R, m) is an integrally closed Noetherian local domain with algebraically closed residue field R/m and the associated graded ring $gr_m R$ is an integrally closed domain.
Let I be a complete m-primary ideal of R and let $v_1, \ldots, v_n$ be the Rees valuations of I. For every $v_i \neq ord_R$, there is a complete m-primary ideal I_i of R such that I_i is quasi-one-fibered (i.e. I_i has exactly one Rees valuation different from ord_R) and the degree coefficient of I_i with respect to v_i is $d(I_i, v_i) = 1$. We discuss some results about complete quasi-one-fibered ideals in 2-dimensional Muhly local domains and we describe a class of projectively full ideals.

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Enlargements of rings

Kristo Väljako

In my talk I will introduce results from the article [1], which I co-authored with prof. V. Laan. There we introduce enlargements of rings as additive analogues of enlargements of semigroups, here we consider additive rings that are not necessarily unital. For example, a full matrix ring over an idempotent ring is an enlargement of that ring. As our main result we proved that two idempotent rings are Morita equivalent if and only if they have a joint enlargement. We also give a necessary and sufficient condition for a ring with left local units to be Morita equivalent to a ring with identity. That condition means that a ring is an enlargement of its local subring.

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[1] Laan, Valdis; Väljako, Kristo, Enlargements of rings, Communications in Algebra, 2021, 1764 – 1772, 49, 4

Integer-valued Skew Polynomials

Nicholas Werner

For an integral domain D with field of fractions K, the ring over integer-valued polynomials on D is $\mathrm{Int}(D) = \{f \in K[x] \mid f(D) \in D\}$. In this talk, we will discuss how to construct generalizations of $\mathrm{Int}(D)$ by using skew polynomials. Given an automorphism σ of K, the skew polynomial ring $K[x;\sigma]$ consists of polynomials with coefficients from K, and with multiplication given by $xa = \sigma(a)x$ for all $a \in K$. We define

$$\mathrm{Int}(D;\sigma) = \{f \in K[x;\sigma] \mid f(D) \in D\},$$

which is the set of integer-valued skew polynomials on D. When σ is not the identity, $K[x;\sigma]$ is noncommutative and evaluation behaves differently than it does for ordinary polynomials. Despite these difficulties, we will show that $\mathrm{Int}(D;\sigma)$ has a ring structure in many cases. While multiplication in these rings is manifestly noncommutative, we can construct interesting commutative rings of polynomials by considering only those polynomials in $\mathrm{Int}(D;\sigma)$ whose coefficients are fixed by σ. Properties of the above rings that may be discussed in this talk include elements, prime and maximal ideals, chain conditions, and behavior under localization.

This will be a talk at the special session “Integer-valued polynomials”.

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Rigid ideals in one-dimensional rings

Roger Wiegand

More than 25 years ago, Craig Huneke and I made various conjectures on the existence of torsion in tensor products. Here is one such conjecture:

Let I be a non-zero ideal of a one-dimensional Noetherian local domain. If $I \otimes_R I^{-1}$ is torsion-free, then I is principal.

The condition that $I \otimes_R I^{-1}$ is torsion-free just means that the natural surjection $I \otimes_R I^{-1} \to II^{-1}$ (given by multiplication) is an isomorphism. In this context, $I \otimes_R I^*$ is MCM (equivalently, torsion-free) if and only if I is *rigid*, that is $\mathrm{Ext}^1_R(I, I) = 0$.

We show, assuming I is rigid, that I must be principal if R has multiplicitiy at most 8 (at most 10 if R is a complete intersection). Also, if I is rigid and the singularity (R, I) is smoothable, then I must be principal.

This talked is based partly on the paper "Rigid ideals in Gorenstein rings of dimension one", C. Huneke, S. B. Iyengar, and R. Wiegand, Acta Math. Vietnamica **44**, 31–49 (2019).

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[1] Authors: T. H. Freitas, V. H. Jorge Pérez, R. Wiegand and S. Wiegand, Journal: Proc. Amer. Math. Soc. 149 (2021), 1817-1825, MSC (2020): Primary 13D07, DOI: https://doi.org/10.1090/proc/14907, Published electronically: March 3, 2021, MathSciNet review: 4232178

Vanishing of Tor over fiber product rings

Sylvia Wiegand

Let (S, m, k) and (T, n, k) be local rings, and let R denote their fiber product ring over their common residue field k. Inspired by work of Nasseh and Sather-Wagstaff, we explore consequences of vanishing of $\mathrm{Tor}_i^R(M, N)$ for various values of i, where M and N are finitely generated R-modules. This is joint work with T. H. Freitas, V. H. Jorge Pérez and R. Wiegand [1].

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[1] T. H. Freitas, V. H. Jorge Pérez, R. Wiegand and S. Wiegand, Proc. Amer. Math. Soc. 149 (2021), 1817-1825, MSC (2020): Primary 13D07, DOI: https://doi.org/10.1090/proc/14907, Published electronically: March 3, 2021, MathSciNet review: 4232178

Absolute irreducibility of the binomial polynomials

Daniel Windisch

In this talk, a result on the factorization behaviour of the binomial polynomials $\binom{x}{n} = \frac{x(x-1)\cdots(x-n+1)}{n!}$ and their powers in the ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$ is presented. It was shown by Cahen and Chabert that the binomial polynomials are irreducible elements in $\mathrm{Int}(\mathbb{Z})$. However, the factorization behaviour of their powers has not yet been fully understood. We fill this gap and show that the binomial polynomials are absolutely irreducible in $\mathrm{Int}(\mathbb{Z})$, that is, $\binom{x}{n}^m$ factors uniquely into irreducible elements in $\mathrm{Int}(\mathbb{Z})$ for all $m \in \mathbb{N}$.
This is joint work with R. Rissner.
This will be a talk at the special session "Integer-valued polynomials".

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Modules with the Exchange Property

Mohamed F. Yousif

A right R-module M is said to satisfy the (full) exchange property if for any two direct sum decompositions $L = M \oplus N = \oplus_{i \in I} N_i$, there exist submodules $K_i \subseteq N_i$ such that $L = M \oplus (\oplus_{i \in I} K_i)$. If this holds only for $|I| < \infty$, then M is said to satisfy the finite exchange property. A ring R for which R_R has the finite exchange property is called an exchange ring. The exchange property is of importance because it provides a way to build isomorphic refinements of different direct sum decompositions, which is precisely what is needed to prove the famous Krull-Schmidt-Remak-Azumaya Theorem. Therefore, one source of motivation for studying the exchange property stems from their connection to the Krull-Schmidt-Remak-Azumaya Theorem.

There is an interesting link between exchange rings and topology. For example, if $C(X)$ is the ring of continuous real valued functions on a topological space X, then a result of Warfield asserts that every projective $C(X)$-module is a direct sum of finitely generated modules iff $C(X)$ is an exchange ring iff X is strongly zero-dimensional. Here, a strongly zero-dimensional space is a Tychonoff space whose Čech-Stone compactification βX is totally disconnected. In the specific case of $C(X)$, being strongly zero-dimensional means any two elements which generate the same principal ideal are associates (i.e. one is a multiple of the other by a unit).

It is an open question due to Crawley and Jónsson whether the finite exchange property always implies the full exchange property. This question was provided a positive answer for modules with indecomposable decompositions by Zimmermann-Huisgen & Zimmermann, for quasi-injective modules by L. Fuchs, for auto-invariant Modules by Guil Assensio and Srivastava, and for Square-free modules by P. Nielsen. In this talk we unify the results on quasi-injective, auto-invariant, and square-free, in one single theorem, and provide new classes of modules, where the finite exchange implies the full exchange.

This will be a talk at the special session "Topological methods in ring theory".

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A reciprocity on finite abelian groups involving zero-sum sequences

Hanbin Zhang

In this talk, we shall present a reciprocity on finite abelian groups involving zero-sum sequences. Let G and H be finite abelian groups with $(|G|, |H|) = 1$. For any positive integer m, let $\mathsf{M}(G, m)$ denote the set of all zero-sum sequences over G of length m. We have the following reciprocity

$$|\mathsf{M}(G, |H|)| = |\mathsf{M}(H, |G|)|.$$

Moreover, we provide a combinatorial interpretation of this reciprocity using ideas from rational Catalan combinatorics. Some related results will also be discussed. This is a joint work with Dongchun Han [1].

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[1] D. Han and H. Zhang, *A Reciprocity on Finite Abelian Groups Involving Zero-Sum Sequences*, SIAM J. Discrete Math., 2021, 35(2):1077 – 1095.

A realization result for systems of sets of lengths

Qinghai Zhong

Let $\mathcal{L}^*$ be a family of finite subsets of $\mathbb{N}_0$ having the following properties.

(a) $\{0\}, \{1\} \in \mathcal{L}^*$ and all other sets of $\mathcal{L}^*$ lie in $\mathbb{N}_{\geq 2}$.

(b) If $L_1, L_2 \in \mathcal{L}^*$, then the sumset $L_1 + L_2 \in \mathcal{L}^*$.

We show that there is a Dedekind domain D whose system of sets of lengths equals $\mathcal{L}^*$.

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